



**Maynooth  
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National University  
of Ireland Maynooth

# Applicable Optics & Sampling Schemes

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# Contents

- Optical design issues
- Optical design, modelling issues and developments
- Horn arrays, packing and sampling
- Single/dual polarisation pixels, Quasioptical LO injection with phase gratings (high optical efficiency required).
- Non traditional telescope designs (wide field of view)
- Number of pixels and field of view (Space mission parameters from literature suggest)
  - $D \sim 10\text{m}$  for sensitivity  $\Rightarrow$  10000 pixel array suggests  $\sim 0.8\text{-}1^\circ$  FOV
  - Might require adopted telescope designs and potentially non standard conic designs (okay with optical analysis tools)

# Generic THz telescope optics

Electromagnetic  
Modelling  
(Waveguides)

Quasi-optical  
Modelling  
Techniques

Physical  
Optics (PO)

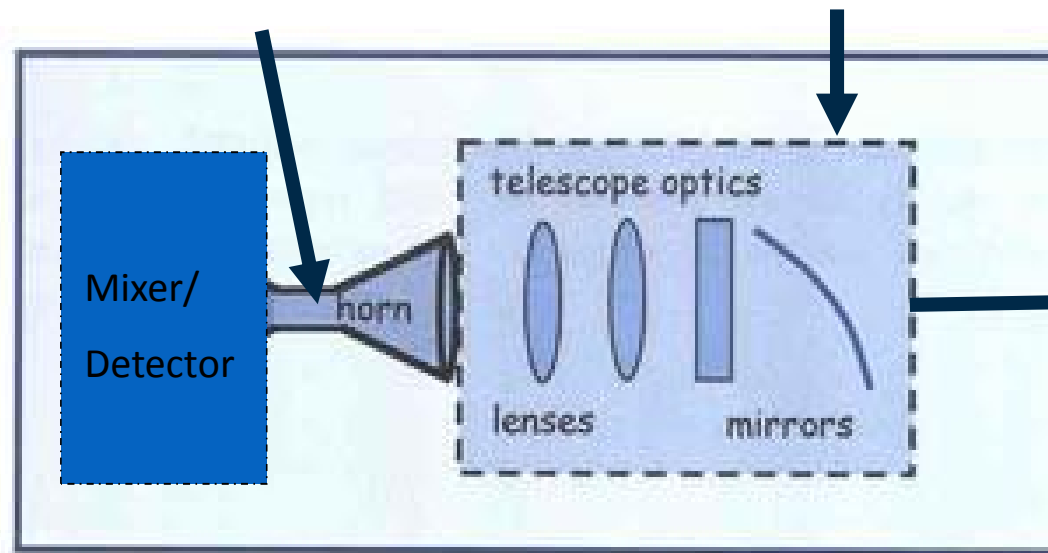


Figure 1. Schematic of a quasi-optical telescope system

Antenna  $F$  number

Intermediate Optics to  
match antenna to telescope

Telescope  $F$  number

# Optical analysis techniques

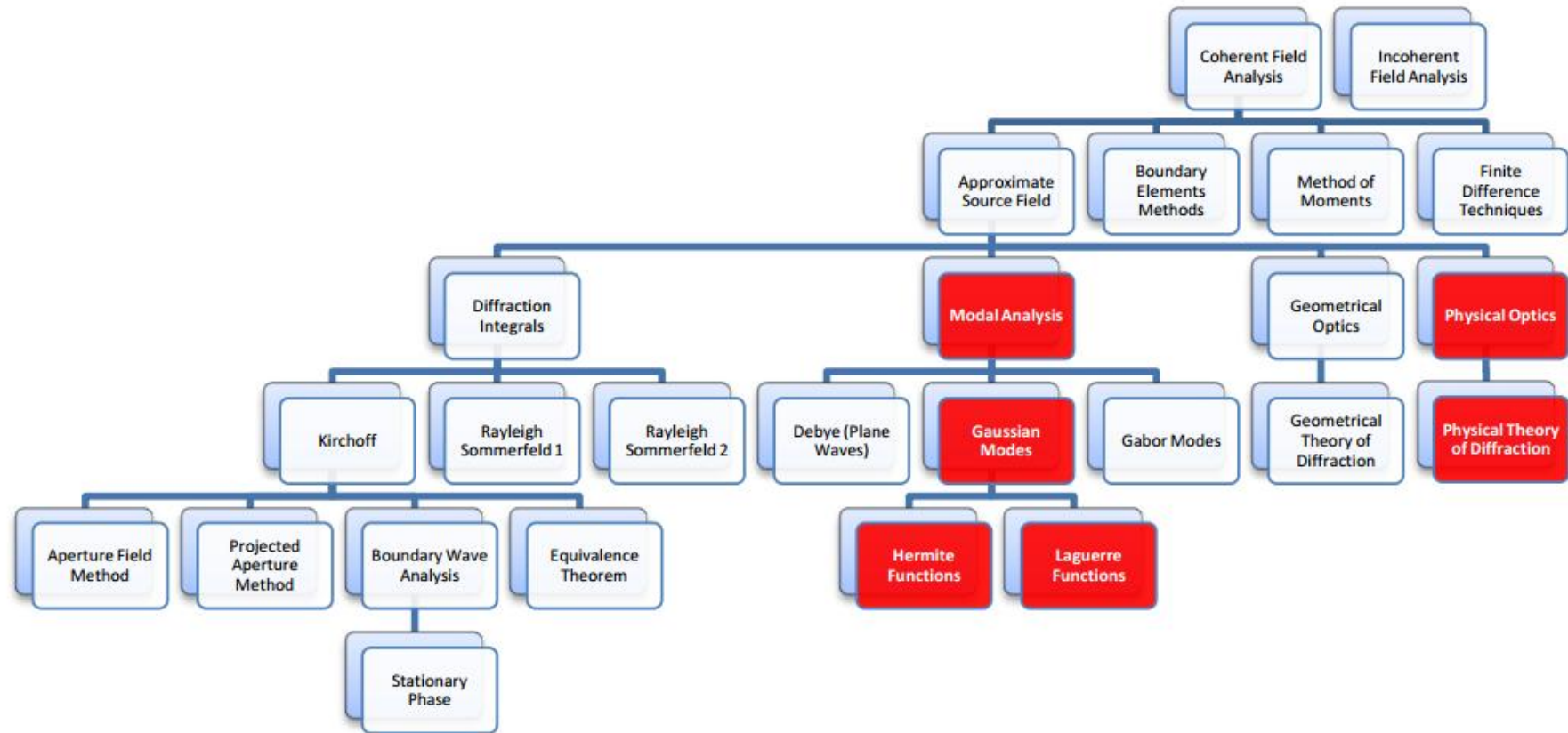
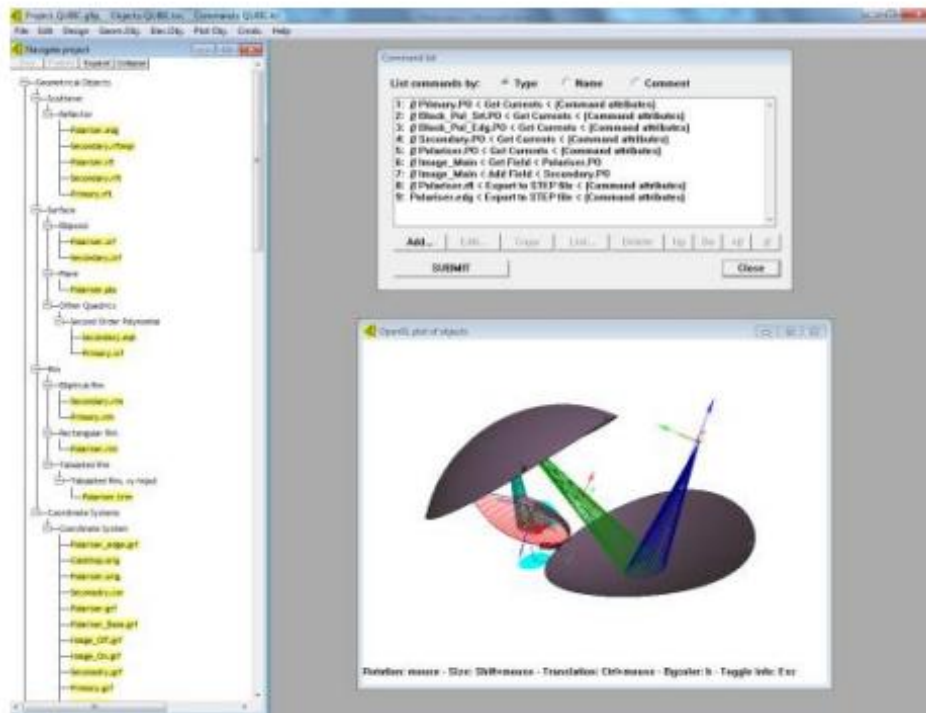


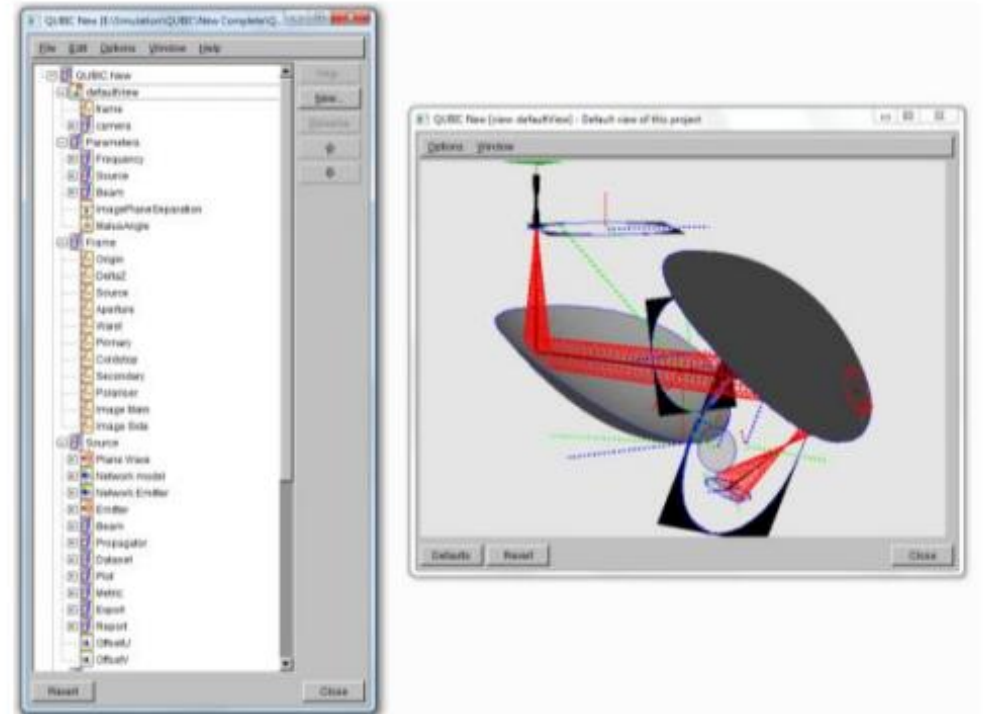
Figure 2-1 - Hierarchy of optical analysis techniques applied to long-wavelength systems

# PO & analysis software

Ticra's GRASP9/GRASP10



The Maynooth Optics Design and Analysis Lab (known as *MODAL*) project





# GBMA to model off axis optics.

- Modal (GBM) can also lead to accurate analysis & highly efficient

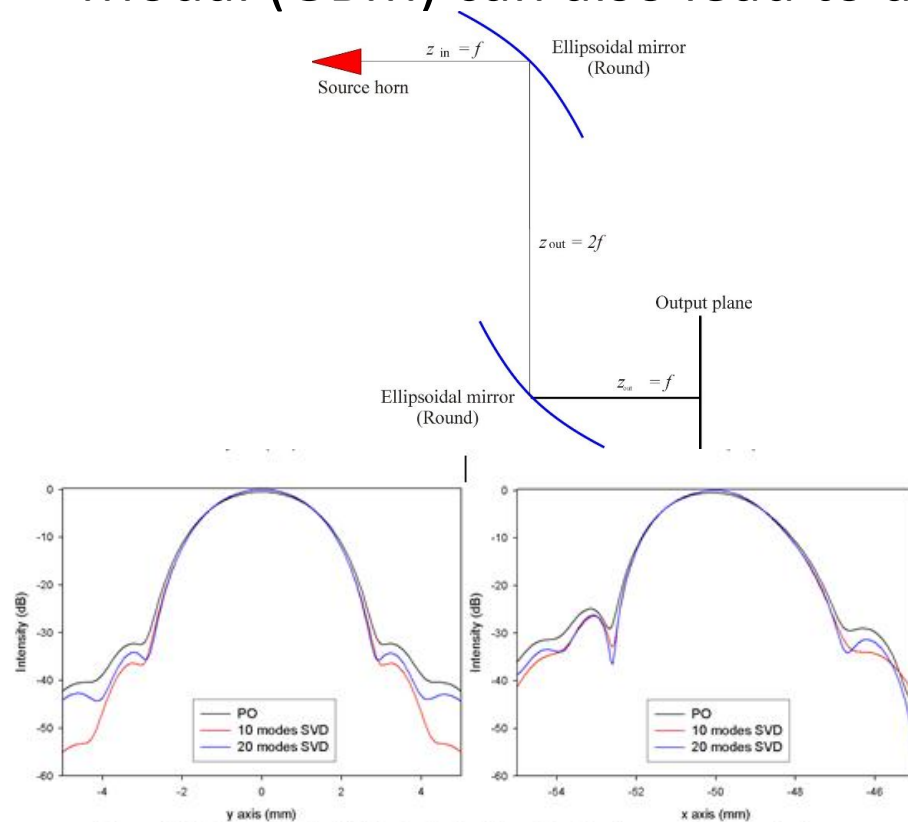
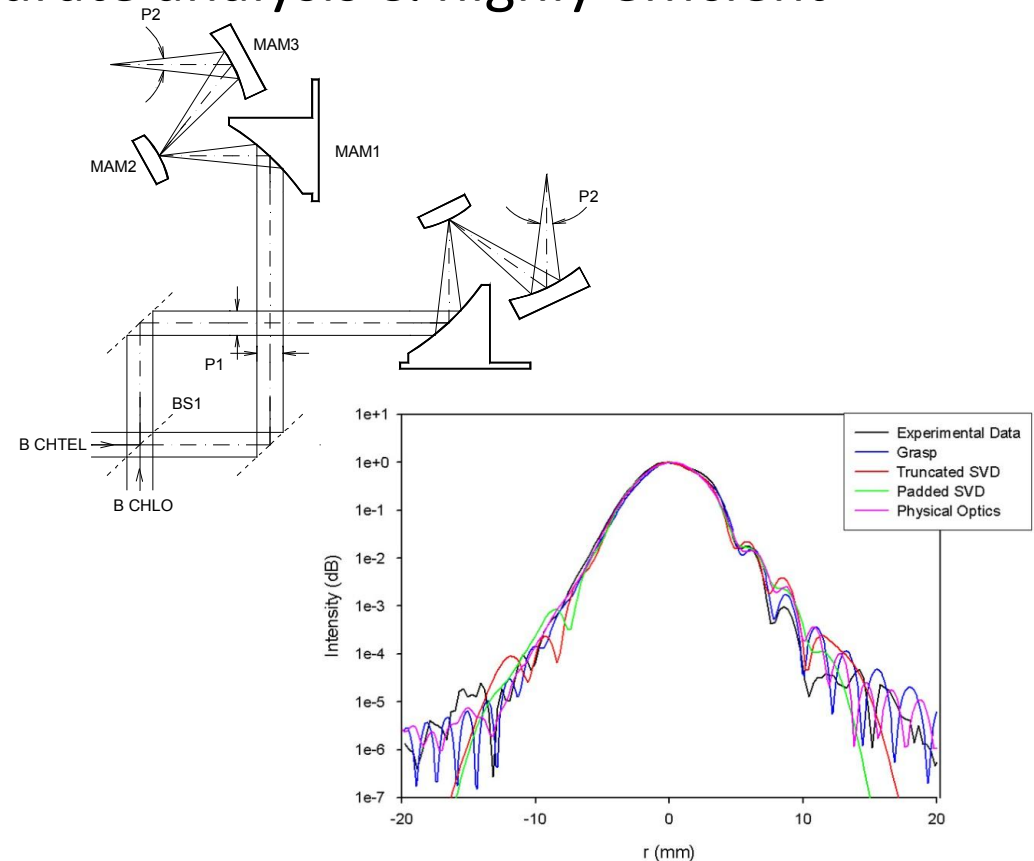
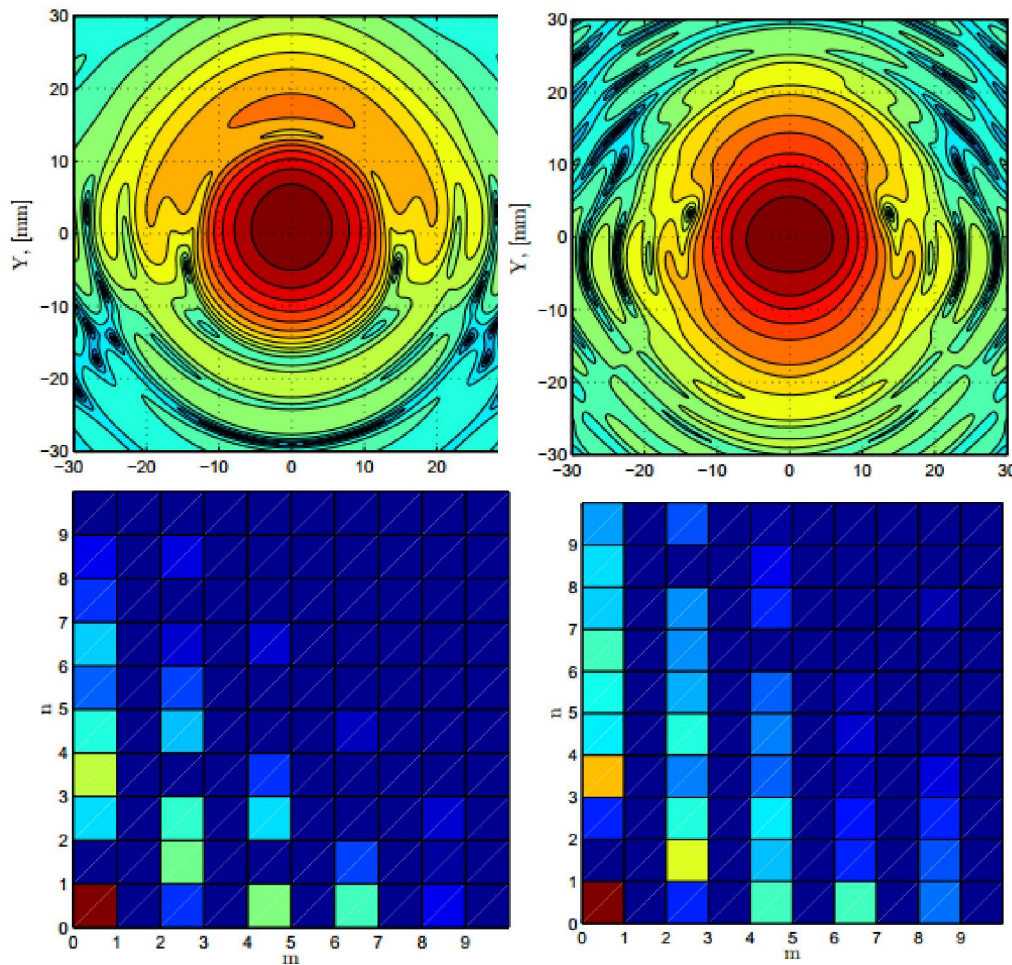


Figure 4.52 Perpendicular field cuts (out of plane/ in plane) across the output plane



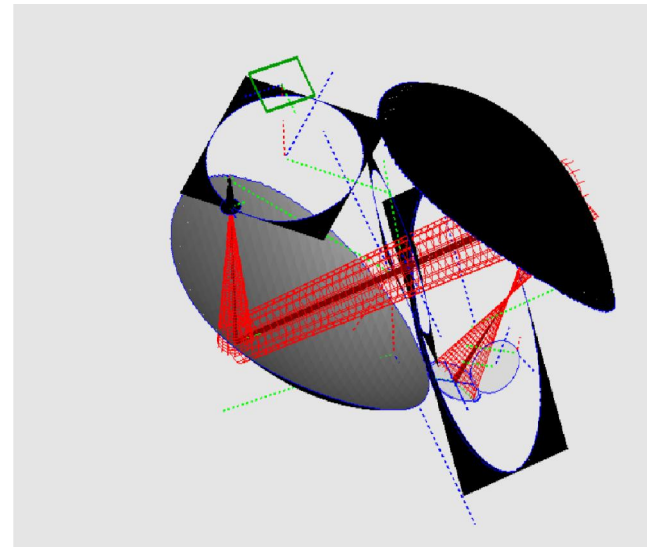
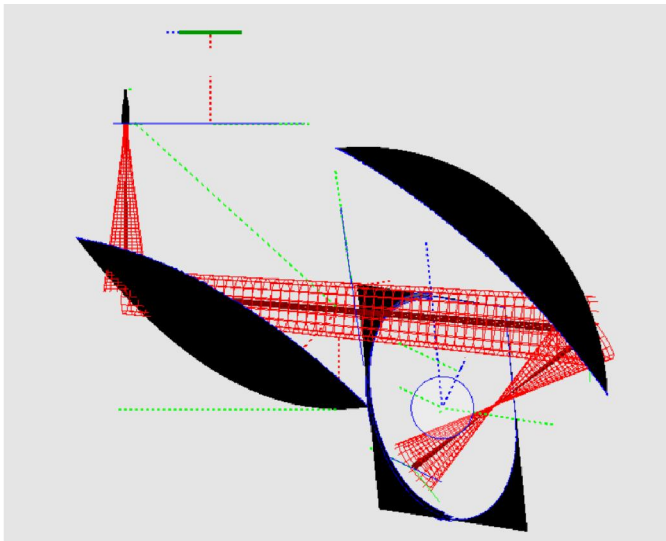
# Using developed code flexibly



- Horns and Eigen fields for multimoded horns (Planck 575, 875GHz horns)
- Multiple reflections.
- Novel optical components

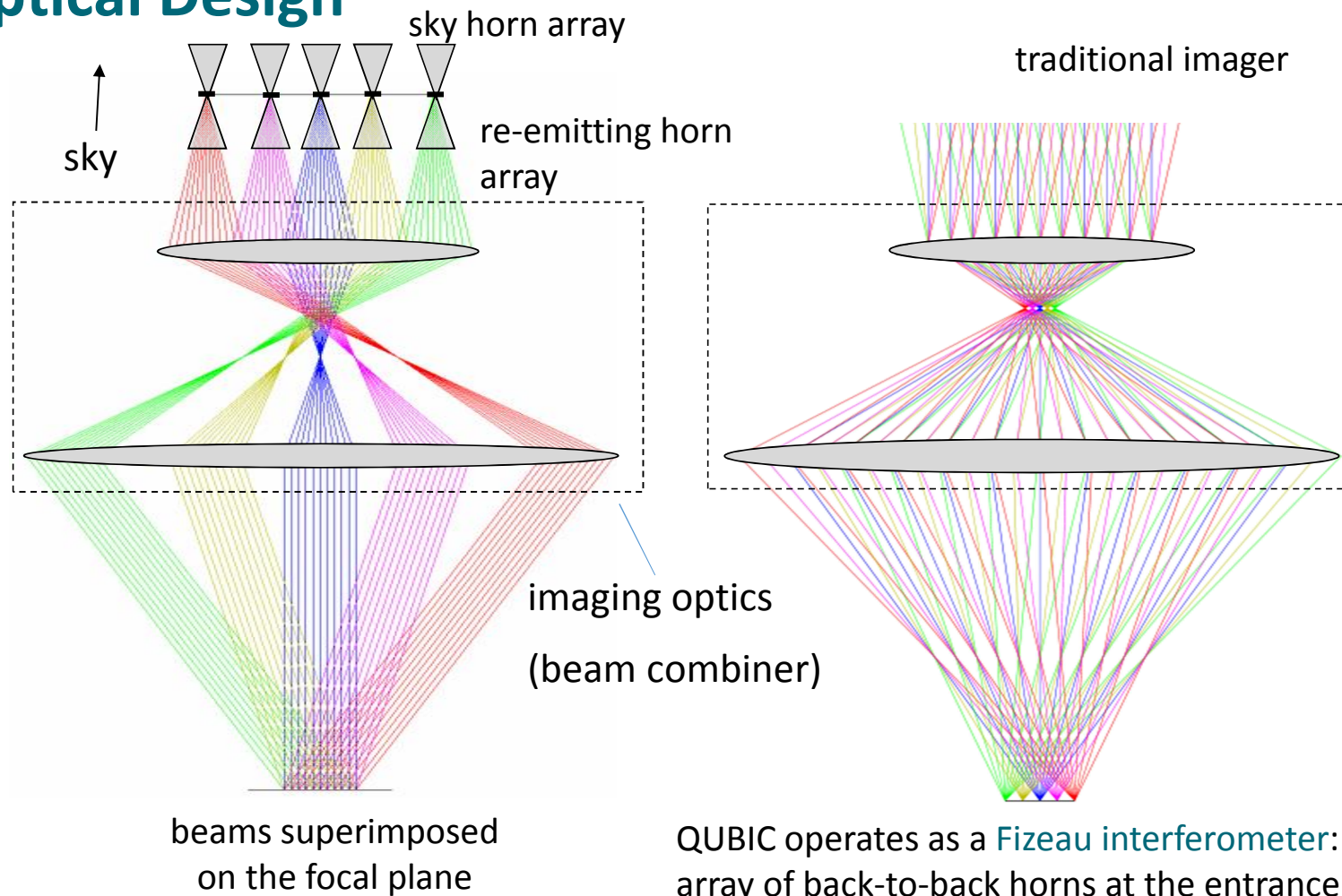
# MODAL and QUBIC

- MODAL (Maynooth Optical Design and Analysis Laboratory)
- Uses Physical Optics and GBM
- Verified against GRASP

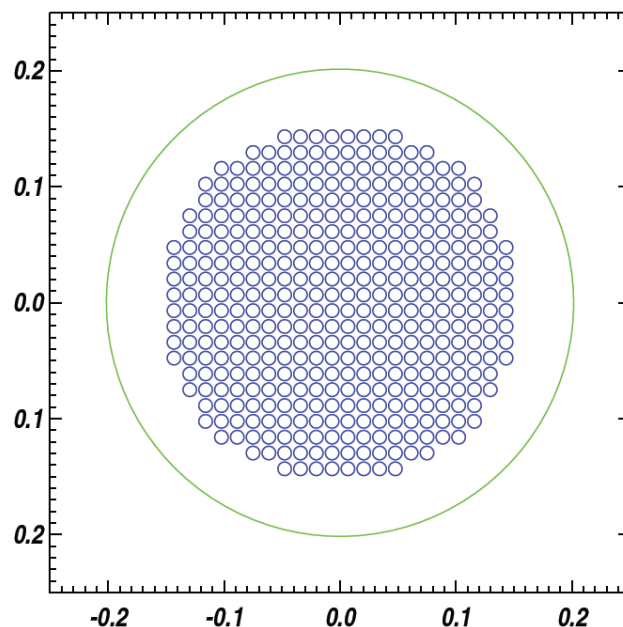




# Optical Design



QUBIC operates as a [Fizeau interferometer](#): the beams from an array of back-to-back horns at the entrance aperture are superimposed on a focal plane by the beam combiner.



Under typical conditions in astronomy, the complex visibility measured in the uv plane and the intensity distribution of the sky are a Fourier pair.

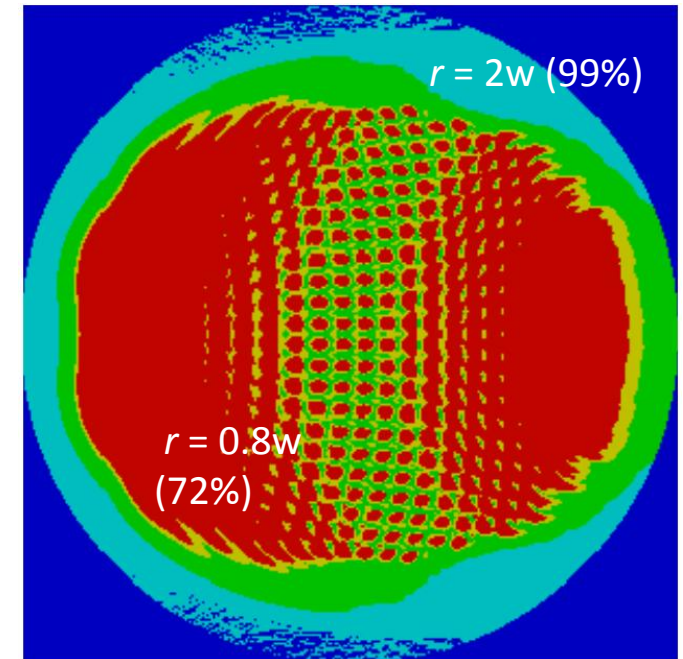
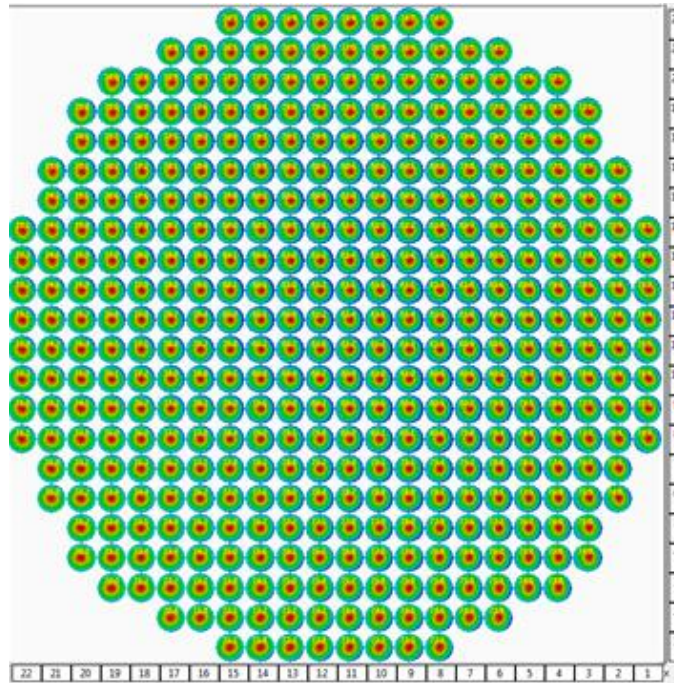
*Focal plane image for one baseline (pair of horns)*

The complex fringe visibility could be measured from this image but QUBIC will be used as a synthetic imager, observing the fringes from all baselines simultaneously.

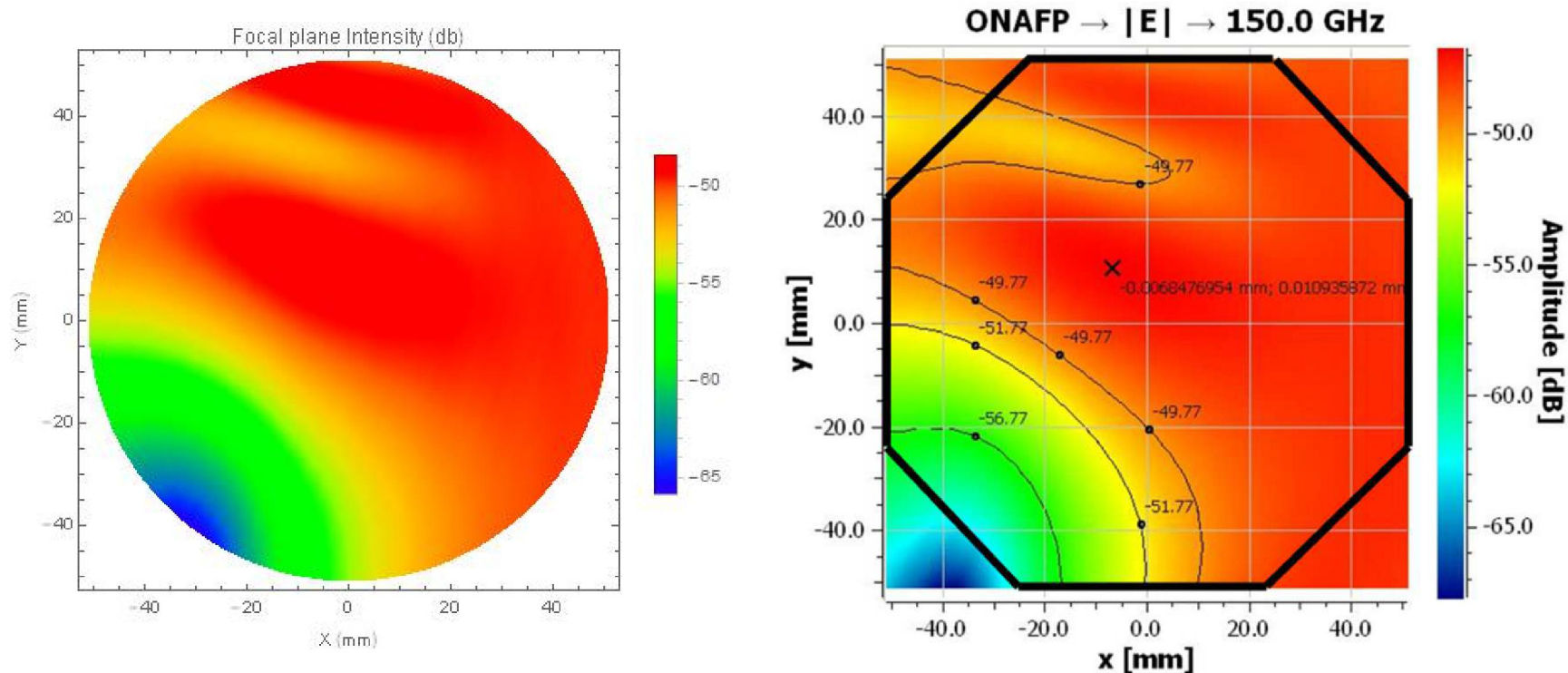
**The combined fringe pattern is simply an image of the sky convolved with the synthesised beam of the instrument - systematics.**

# Horn array

- QUBIC has a 400 horn array
- Modal propagates each horn through the system individually
- This image is the beam pattern on the focal plane, secondary mirror of each horn positioned corresponding to the location of the horn in the array



# Comparison with GRASP

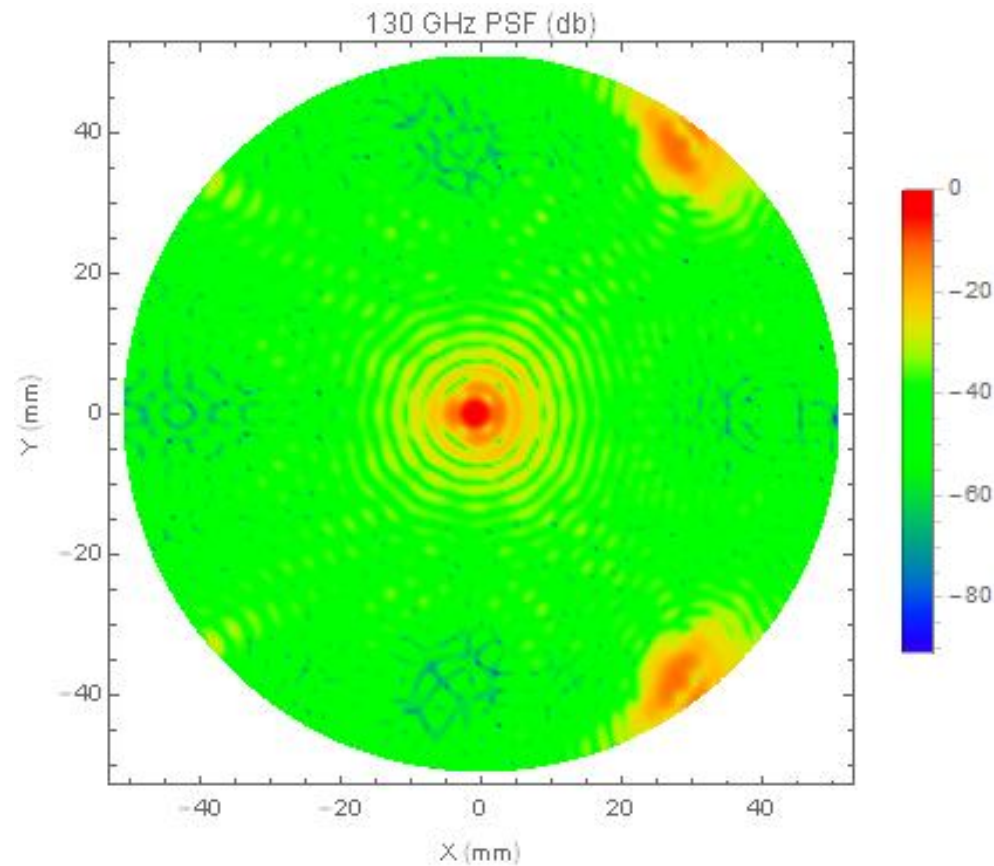


- Simulated results in MODAL (left) matched the results obtained from GRASP (right)



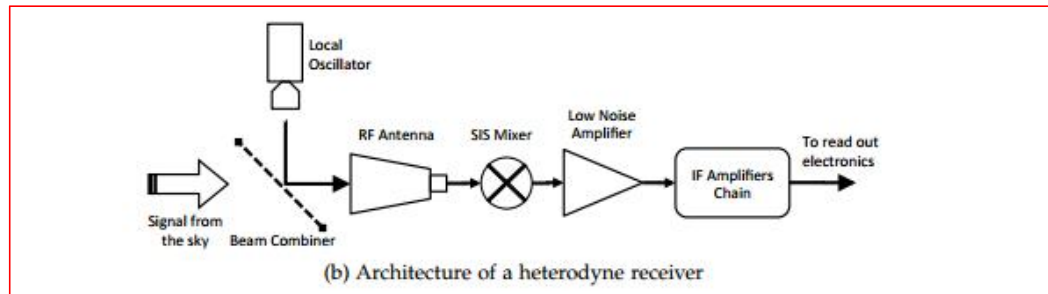
# PSF

- The data from each of the horns can be combined to get the PSF on the focal plane

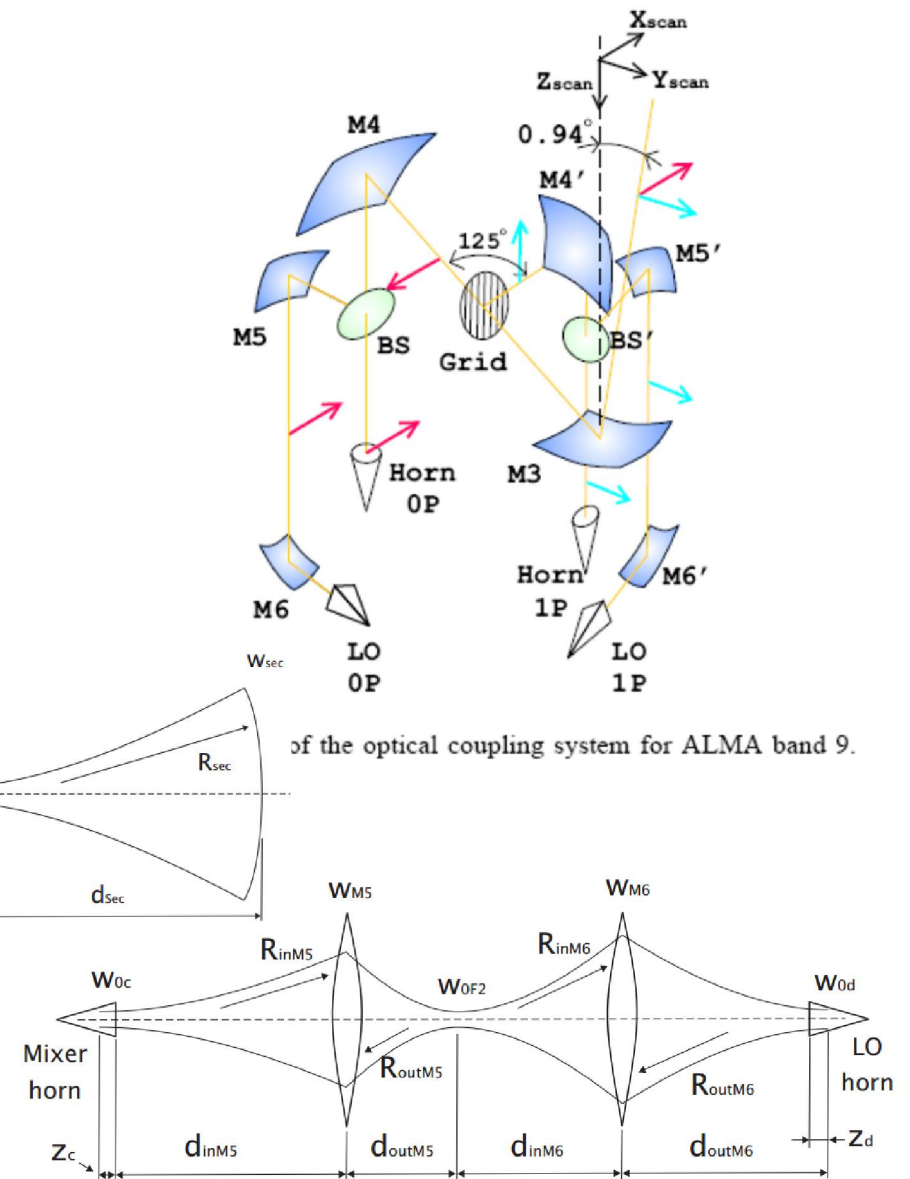
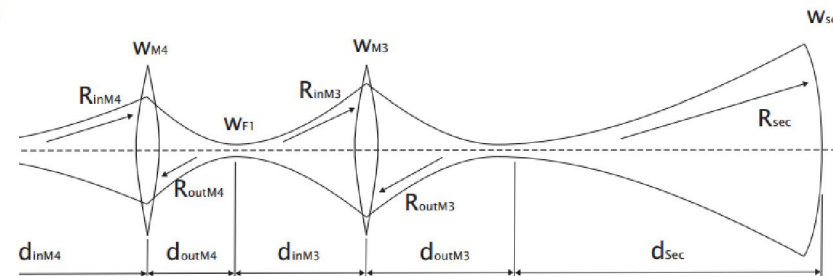
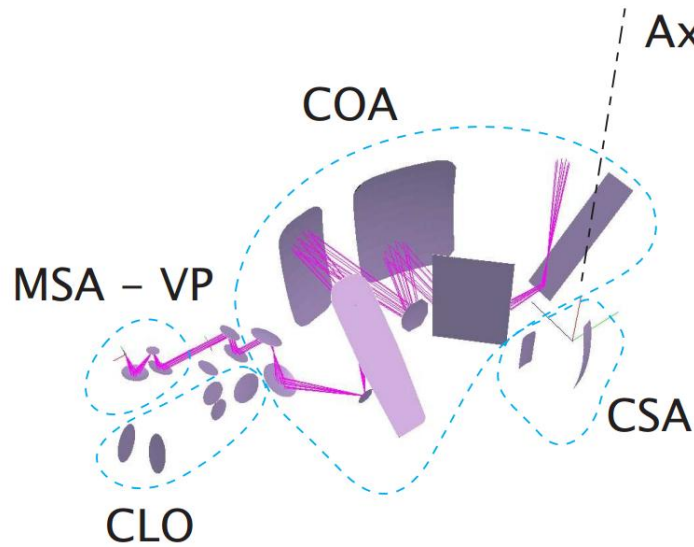




# Heterodyne receivers



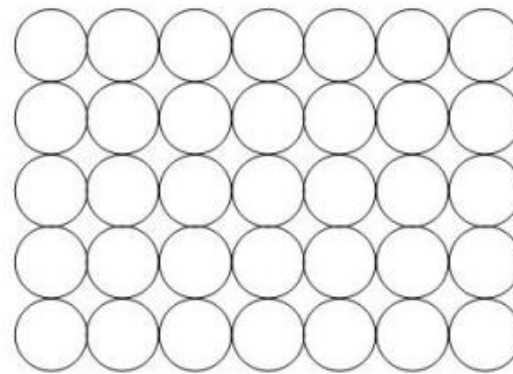
## Telescope



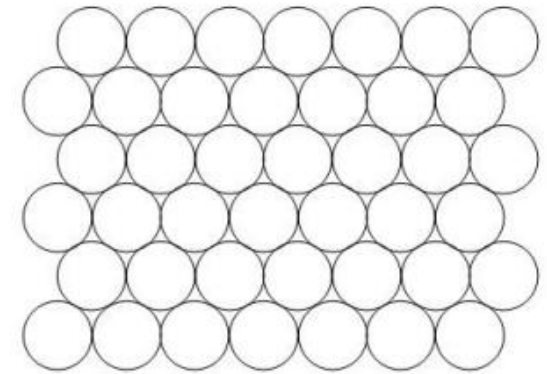
# Scanning & sampling

- Close packed horn ( $\sim 1-3F\lambda$ ) arrays will not fully sample the sky unless the horns are less than Nyquist sampled  $\approx 0.5F\lambda$   
*Griffin et al 2002* highlights 16 telescope pointings for  $2F\lambda$  horn arrays or scanning at an appropriate angle to provide overlap of beams in area of interest.

- Horns give good bandwidth definition
- Reduces stray light & need for a cold stop
- Well understood
- Paper “The Relative Performance of Filled and Feedhorn-Coupled Focal-plane Architectures”
- [M.J. Griffin](#), [J.J. Bock](#), [W.K. Gear](#)



(a) Square Packing



(b) Hexagonal Packing

DOI: [10.1364/AO.41.006543](https://doi.org/10.1364/AO.41.006543)

# Scanning & sampling

- Using a hexagonal layout, the angular on sky sampling rate is

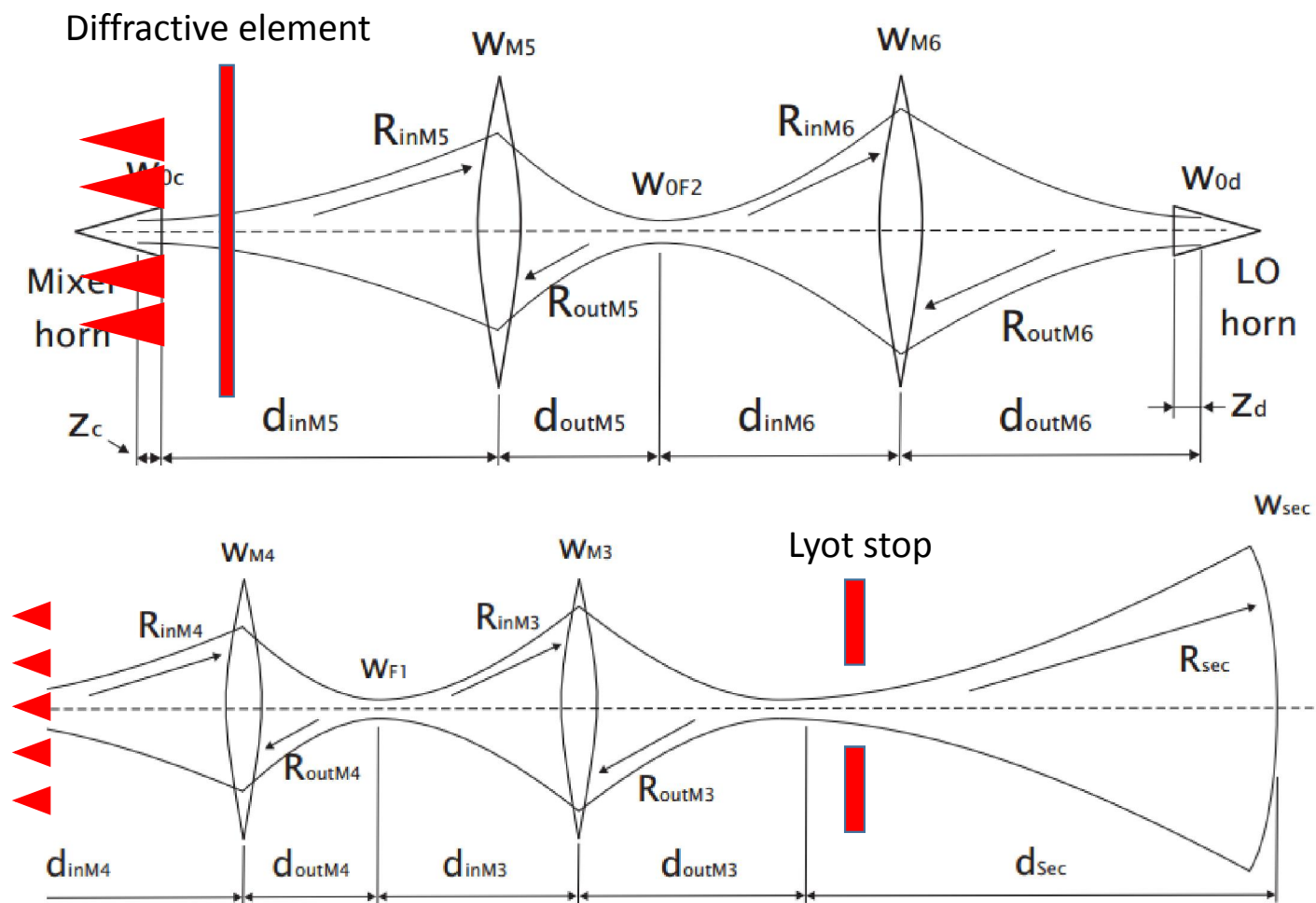
$$\Delta\theta_{Ny} \approx \frac{\theta_{FWHM}}{\sqrt{3}} \approx \frac{\lambda}{\sqrt{3}D}$$

- where  $D$  is telescope diameter. In the focal plane then

$$\Delta x_{Ny} \approx \frac{\lambda f}{\sqrt{3}D}$$

- where  $f$  is the equivalent focal length of the telescope Assume the -3 dB beam width,  $\theta_{FWHM}$ , is assumed to be  $\lambda/D$  on-sky and  $\lambda f/D$  on focal plane.

# Equivalent optical train sky & LO paths





European Space Agency

Presentation entitled: " **Next Generation Sub-millimetre Wave Focal Plane Array Coupling Concepts – An ESA TRP project to develop multichroic focal plane pixels for future CMB polarisation experiments.** "

ITT AO/1-7393/12/NL/MH

## Next Generation Sub-millimetre Wave Focal Plane Array Coupling Concepts

*<sup>1</sup>Trappe N.; <sup>2</sup>Bucher M.; <sup>3</sup>De Barnardis P.; <sup>2</sup>Delabrouille J.; <sup>4</sup>Deo P.; <sup>3</sup>De Petris M.; <sup>1</sup>Doherty S.; <sup>2</sup>Ghribi A.; <sup>1</sup>Gradziel M.; <sup>5</sup>Kuzmin L.; <sup>4</sup>Maffei B.; <sup>5</sup>Mahashabde S.; <sup>3</sup>Masi S.; <sup>1</sup>Murphy J.A.; <sup>4</sup>Noviello F.; <sup>1</sup>O'Sullivan C.; <sup>3</sup>Pagano L.; <sup>3</sup>Piacentini F.; <sup>2</sup>Piat M.; <sup>6</sup>Pisano G.; <sup>4</sup>Robinson M.; <sup>2</sup>Stomp R.; <sup>2</sup>Tartari A  
<sup>1</sup>Maynooth University, <sup>2</sup>APC Paris, <sup>3</sup>La Sapienza University of Rome, <sup>4</sup>University of Manchester, <sup>5</sup>Chalmers University, <sup>6</sup>Cardiff University*

### Institutes participating

APC Paris Laboratoire de Astroparticule et Cosmology

Cardiff University, UK

Chalmers Technical University Göteborg

La Sapienza, Rome

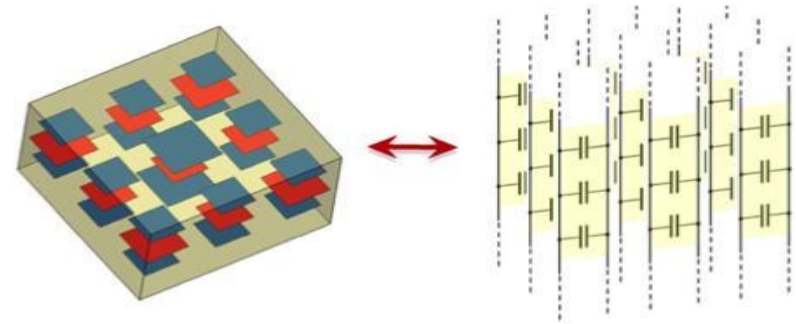
Manchester University, UK

NUI Maynooth, Ireland



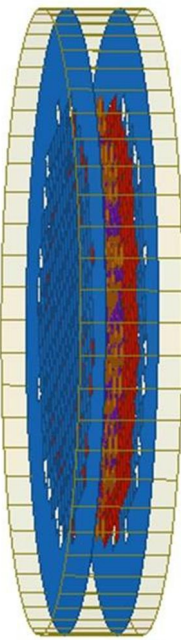


# Planar mesh lens

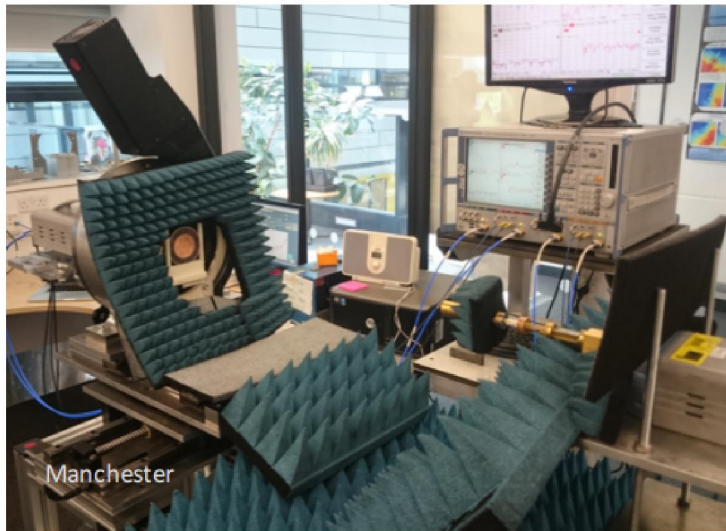


Inhomogeneous metal mesh devices can be modelled with a two-dimensional array of different TLs [Pisano et al, 2013].

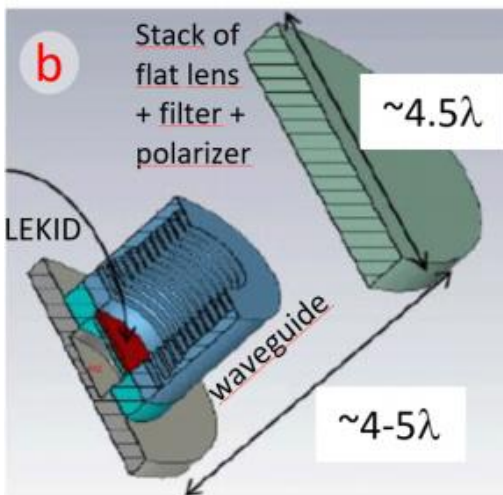
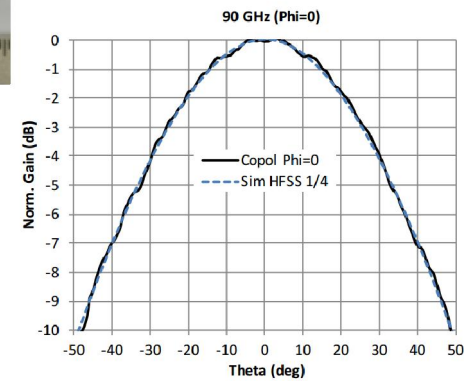
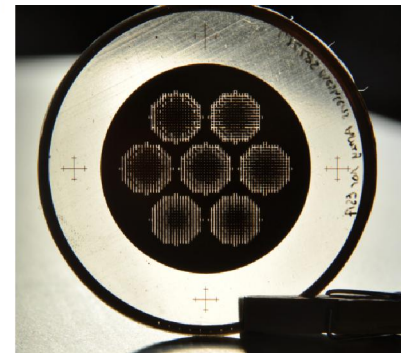
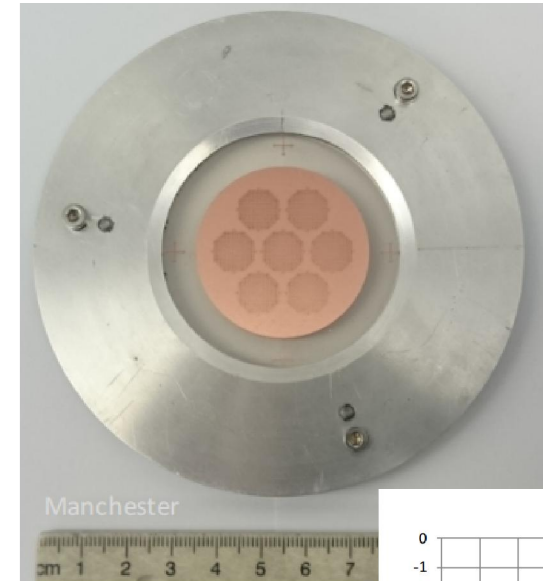
- This option is based on a planar mesh lens
- It is based on an inhomogeneous metal mesh device can be modelled with a two-dimensional array of different TLs [Pisano et al, Applied Optics 52,n.11, (2013)]
- In order to achieve the appropriate transmissions and phase delay across the lens surface, a specifically developed code is used which can optimise more than one thousand TLs to simulate the action of a traditional “thick ” dielectric lens
- This planar thin mesh lens is very robust, and can be used cryogenically as well as in space environments as normal mesh filters.



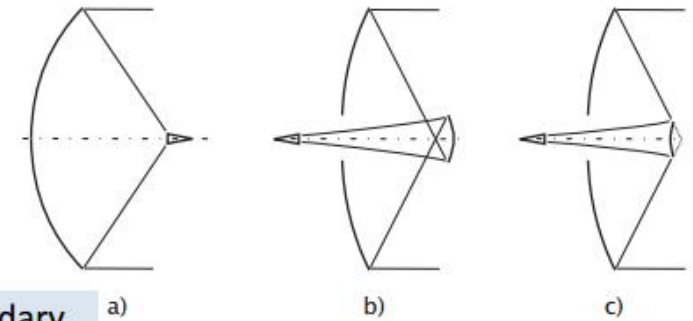
## Manufacture/Measurements at Manchester University



▲ Measurement set-up.

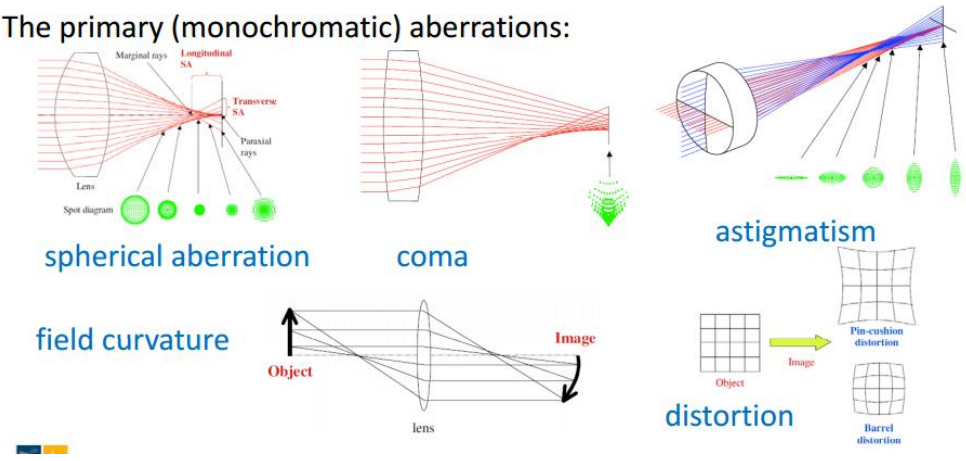


These planar lenslets are illuminated by a waveguide section which feeds a planar OMT for the Low Frequencies, or directly a LEKID in the CMB bands



Classical off-axis Gregorian	primary, confocal elliptical secondary	real exit pupil, small secondary
Classical off-axis Cassegrain	parabolic primary, confocal hyperbolic secondary	compact
Classical Offset Dragonian reflector	parabolic primary, confocal (concave) hyperbolic secondary	good field-of-view
<b>Classical Offset Dragonian reflector, crossed geometry</b>	<b>parabolic primary, confocal (concave) hyperbolic secondary</b>	<b>can be side-fed (SFOC / CATR) or front-fed</b>
Classical offset Inverse Cassegrain	paraboloidal main reflector and a concave ellipsoidal sub reflector	reduced astigmatism
Aplanatic Gregorian	Slightly ellipsoidal primary, confocal (convex) elliptical secondary	spherical aberration & coma corrected, reduced astigmatism
Aplanatic Cassegrain	Slightly hyperbolic primary, hyperbolic secondary	Ritchey-Chrétien , spherical aberration & coma corrected, larger astigmatism

The primary (monochromatic) aberrations:



Classical offset Cassegrain and Gregorian dual reflectors are currently in wide use in many high-performance applications. However, the Gregorian does not have a large usable field-of-view and offset Cassegrain have long equivalent focal lengths and relatively large astigmatism. The classical Dragonian antenna employing a paraboloidal main reflector and a concave hyperboloidal subreflector has been shown to produce good performance over wide fields-of-view.

Any 2-reflector system consisting of offset conic sections has an "equivalent paraboloid" with the same effective focal length and aperture field distribution.

By adjusting the relative tilt of the parent conic symmetry axes the equivalent paraboloid can be made to be rotationally symmetric giving low cross-polarisation and low astigmatism.

This configuration, called a compensated system, is said to obey the Mizuguchi-Dragone condition.

A classical compensated off-axis 2-mirror telescope has zero linear astigmatism, and coma (the dominant remaining aberration) identical to that of an on-axis paraboloidal mirror with the same focal ratio.



## Requirement for a COrE-type instrument

From WP1.1

6750 detectors

dual polarisation → 3375 devices

× area of horn ( $2.3 F\lambda$ )

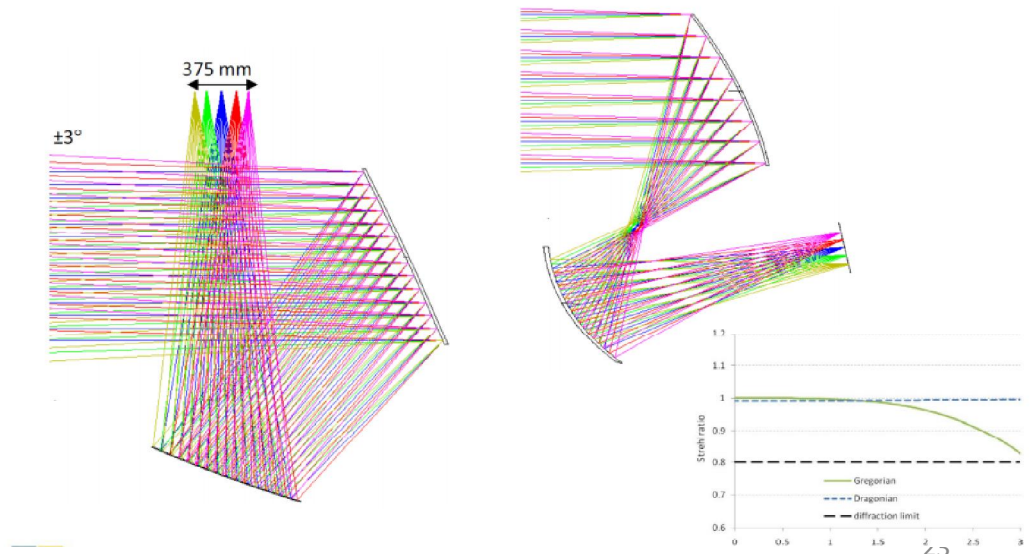
90% filling factor

F/3 requires a focal plane of radius ~375 mm

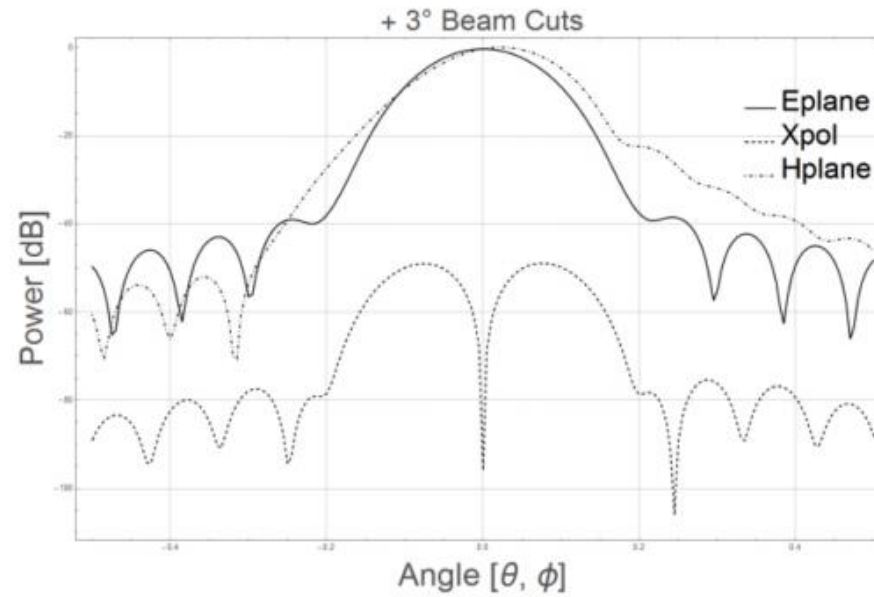
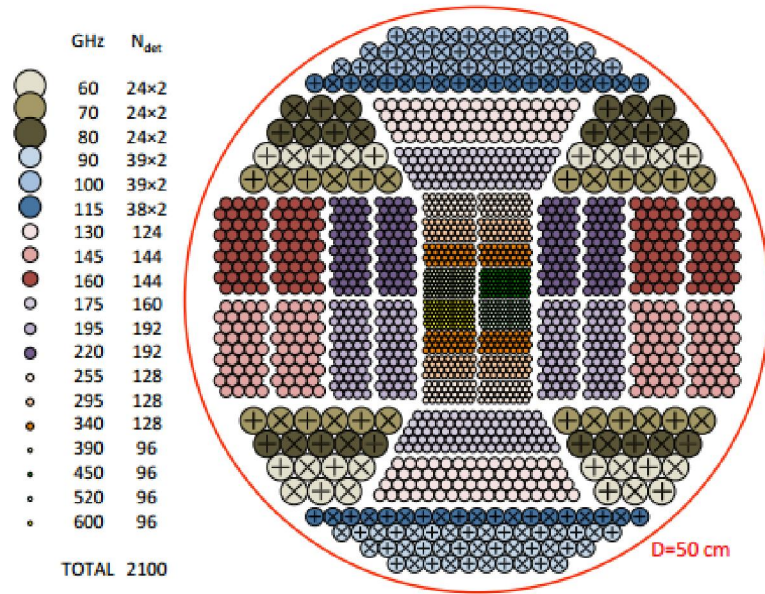
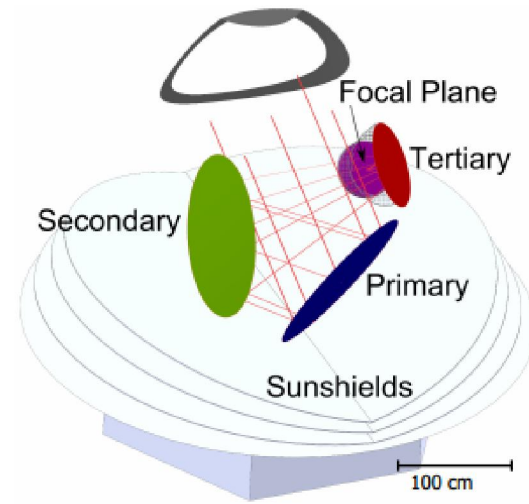
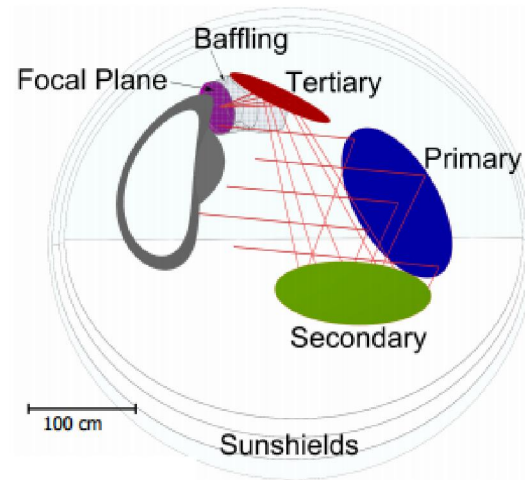
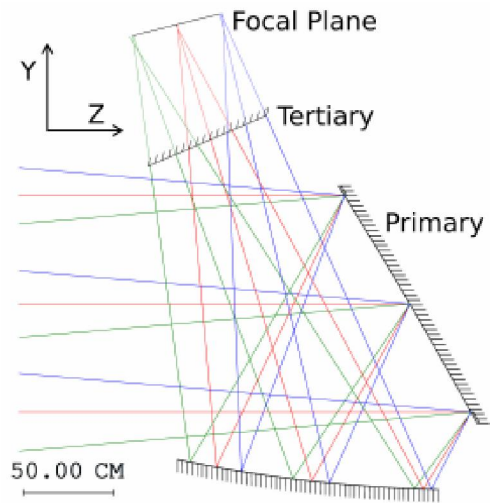
F/2 requires a focal plane of radius ~250 mm

channel GHz	$dv/\nu$	beam arcmin	$N_{\text{det}}$
45	0.333	23.33	10
75	0.2	14	90
105	0.143	10	750
135	0.111	7.78	1000
165	0.091	6.36	1000
195	0.077	5.38	1000
225	0.067	4.67	1000
255	0.059	4.12	300
285	0.053	3.68	300
345	0.13	3.04	300
435	0.103	2.41	200
525	0.086	2	200
615	0.073	1.71	200
705	0.064	1.49	200
825	0.091	1.27	200
<b>TOTAL</b>	-	-	<b>6750</b>

conics arranged in a crossed configuration smaller aberrations

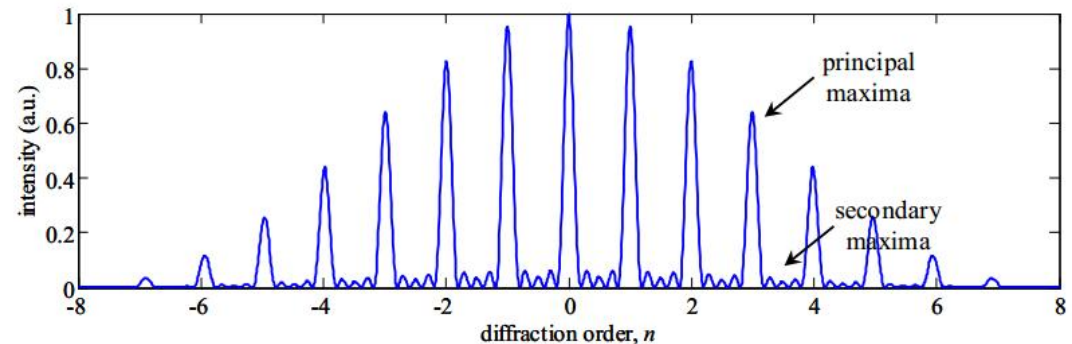






Primary mirror		Secondary mirror	Telescope geometry	
131 cm × 152 cm		125 cm × 146 cm	$D_m$	120 cm
$R_x$	$-1.15 \times 10^3$	$8.29 \times 10^2$	$L_m$	112 cm
$R_y$	$-7.14 \times 10^2$	$8.33 \times 10^2$	$L_s$	264 cm
$k_x$	2.97	-0.574	$h$	765 cm
$k_y$	-3.55	-7.31	$\alpha$	13.8°
$A_{2,r}$	$3.02 \times 10^{-10}$	$1.31 \times 10^{-9}$	$\beta$	90.2°
$A_{3,r}$	$9.54 \times 10^{-18}$	$3.05 \times 10^{-14}$	$\theta_0$	104°
$A_{4,r}$	$-1.63 \times 10^{-22}$	$-8.17 \times 10^{-19}$		
$A_{5,r}$	$-3.49 \times 10^{-34}$	$2.36 \times 10^{-24}$		
$A_{2,p}$	-1.027	0.183		
$A_{3,p}$	-0.255	0.0671		
$A_{4,p}$	-0.281	0.259		
$A_{5,p}$	-0.914	0.669		
Tertiary mirror		104 cm × 74 cm		
Focal ratio, F		2.54 at centre, varies by 5 % across focal plane		
160 GHz DLFOV		$A_x \times E_l = 14.0^\circ \times 12.9^\circ$ , $159 \text{ F}\lambda \times 140 \text{ F}\lambda$		
Focal Surface Location		Center	$\pm A_x$ , edge	$\pm E_l$ , edge
Instrumental polarization (%)		0.06	0.07, 0.05	0.07 0.05
Polarization rotation (°)		0	$\pm 0.6$	0 0

# Diffraction Grating



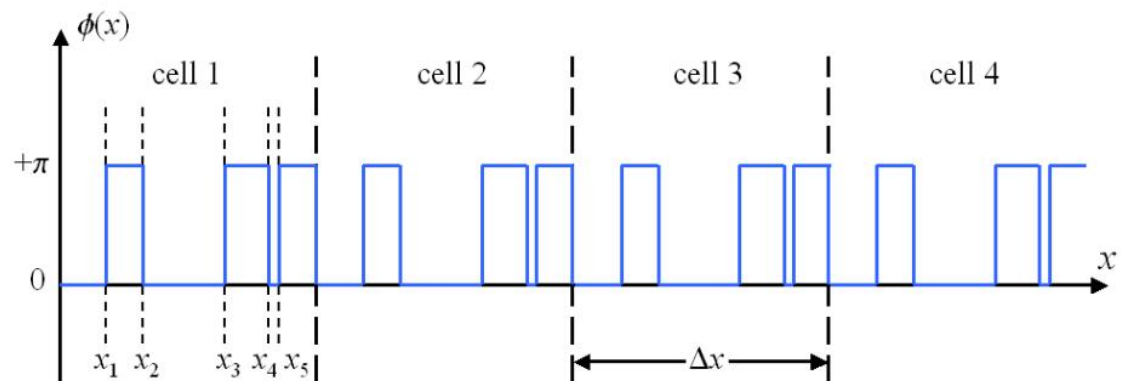
- In the most general terms a diffractive optical element (DOE) is any optical element that imparts a wavefront transformation on an incident wavefront. (reflective or refractive options)
- Used as a beam combiner also in reverse.
- The ratio of aperture  $a$  to separation  $d$  is important in controlling separation and intensity of farfield beams.
- For beam splitting applications not all higher orders are useful. Instead we require limited incident power be redirected into only a small number of the diffraction orders – the signal orders – and other parasitic orders be suppressed.

# Dammann Grating

- A DG is a binary optical element that subjects a beam to a phase-only modulation (0 and  $\pi$  radians), due to relative path length differences imposed by the surface grooves. A delay of one half wavelength corresponds to a phase shift of  $\pi$  radians

$$T_G(u, v) = T_1(u).T_2(v)$$

$$t_G(x, y) = t_1(x)t_2(y) = e^{-i\phi_G(x, y)}$$



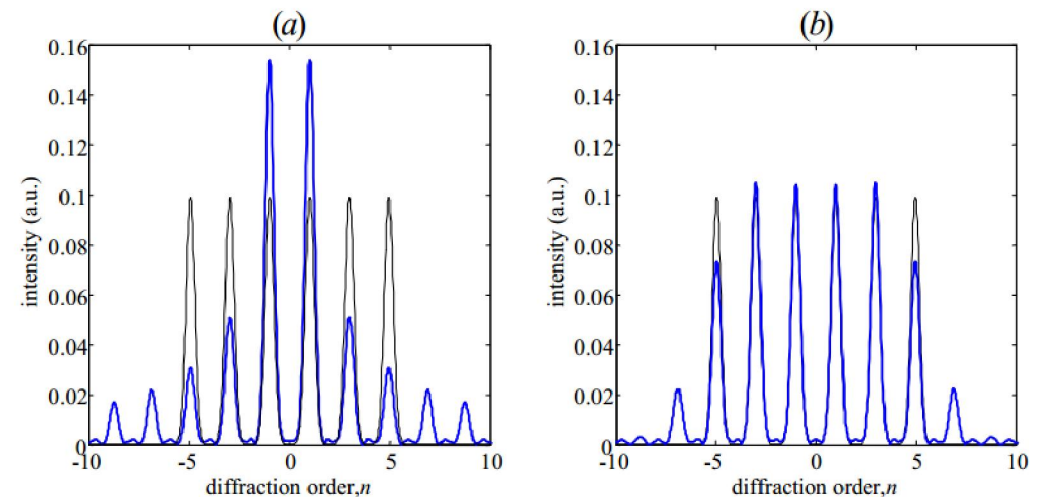
# Dammann Grating

- The diffraction envelope  $T(u,v)$  of a Dammann grating is given by the Fourier transform of the transmission function  $t_{\text{cell}}(x, y)$  of the unit cell. Typically this calculation is performed with a discrete Fourier transform (FT) or GBMA.

$$f(x) = \sum_{i=1}^N f(x_i) \cdot \text{rect}\left(\frac{x-x_i}{\Delta x}\right)$$

$$F(u) = \sum_{i=1}^N f(x_i) \cdot |\Delta x| \text{sinc}(\Delta x u) e^{-i 2\pi x_i u}$$

$$\eta_1 = \frac{\text{radiant flux within signal orders}}{\text{total radiant flux through the grating}} \leq 1$$

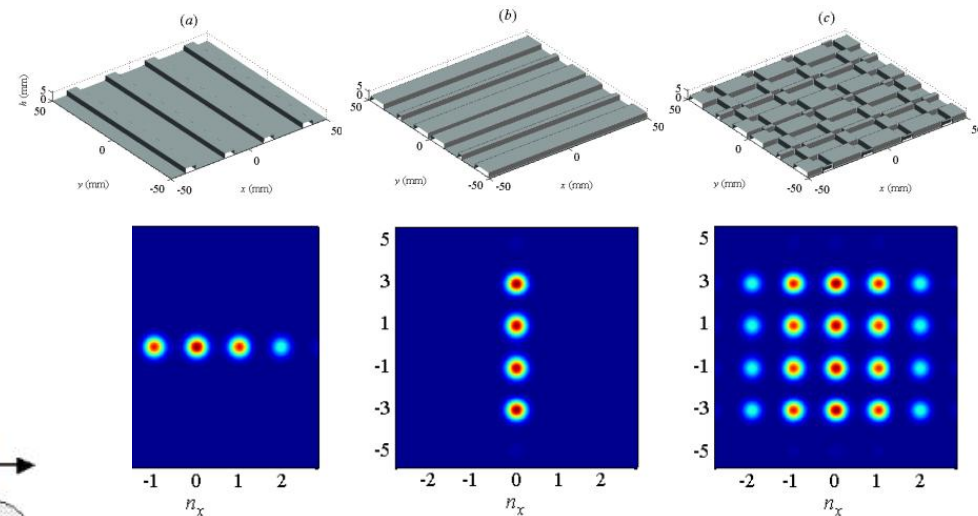
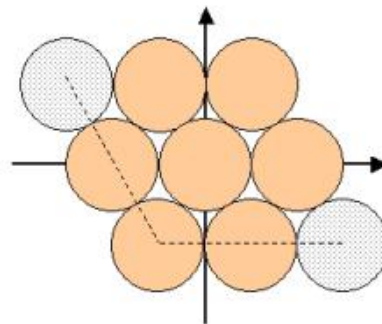
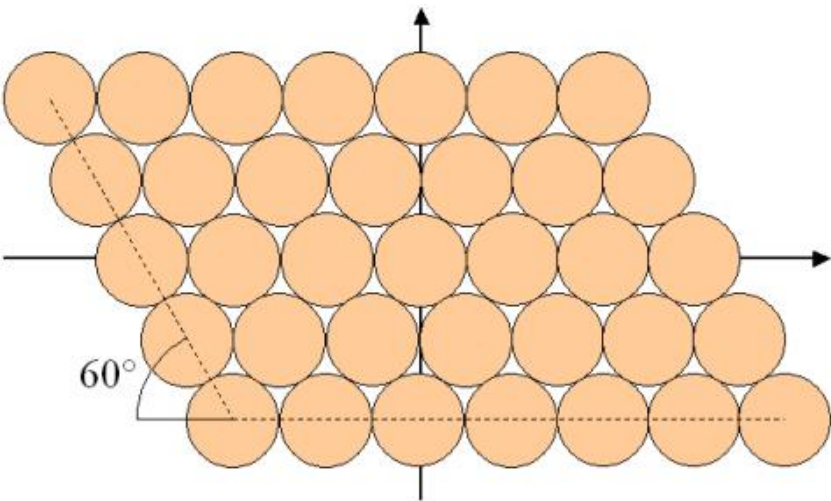


both intensities yield almost identical values of ~93%.

$$\eta_{\text{weight}} = \eta \cdot (1 - \text{MPD}) = \eta \cdot \text{MPU}$$

# Quasioptical Dammann Grating

- Tightly packed array of output beams is produced by overlaying two linear gratings such that their grooves cross at an angle of  $60^\circ$  rather than  $90^\circ$ .





# Reflective/Bandwidth effects

- For a reflective grating a phase modulation  $\phi(x)$  for all  $x_{in}$  a reflective DPE is

$$h(x) = \frac{\phi(x)}{2k_0 \cos(\alpha)}$$

- To compensate for this projection effect, the grating surface profile must be elongated. The grating surface  $h(x)$  is now defined in terms of transverse coordinates

$$x' = \frac{x}{\cos(\alpha_{inc})}$$

- Bandwidth effects - the phase modulation imparted by a DPE at frequency  $\nu \neq \nu_0$

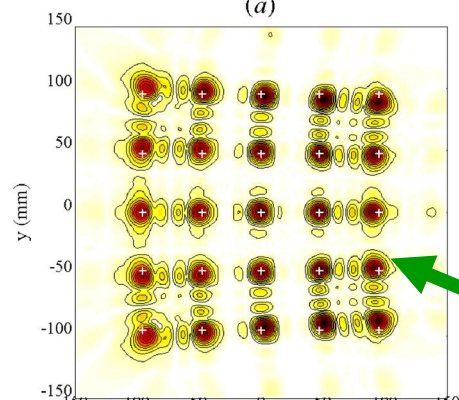
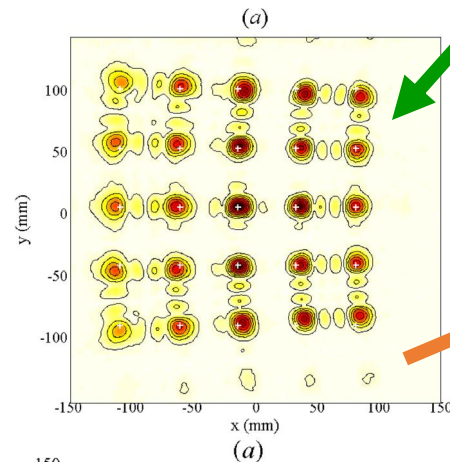
is equal to the design phase modulation  $\phi_0(x)$  scaled by the ratio of  $\nu$  to  $\nu_0$

$$\phi(x) = \phi_0(x) \cdot \frac{k}{k_0} = \phi_0(x) \cdot \frac{\lambda_0}{\lambda} = \phi_0(x) \cdot \frac{\nu}{\nu_0}$$

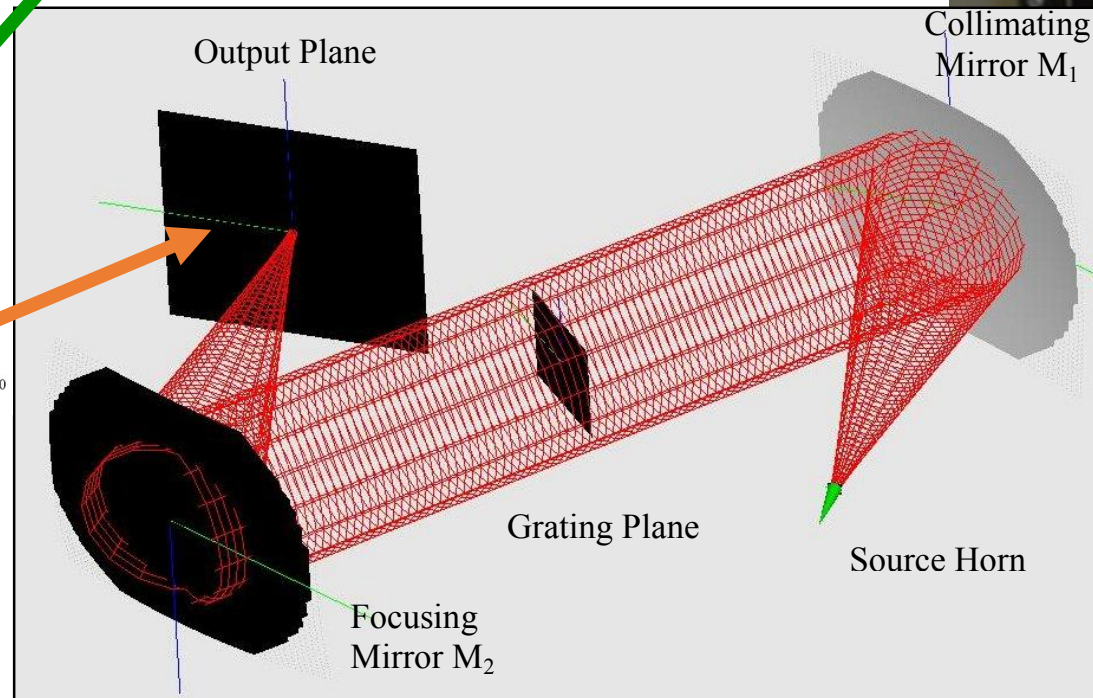
## Example: Dammann Phase Grating

- *Dammann Grating (Binary) -*
- *Analyse off-axis configurations using MODAL.*

• *Measured Pattern*

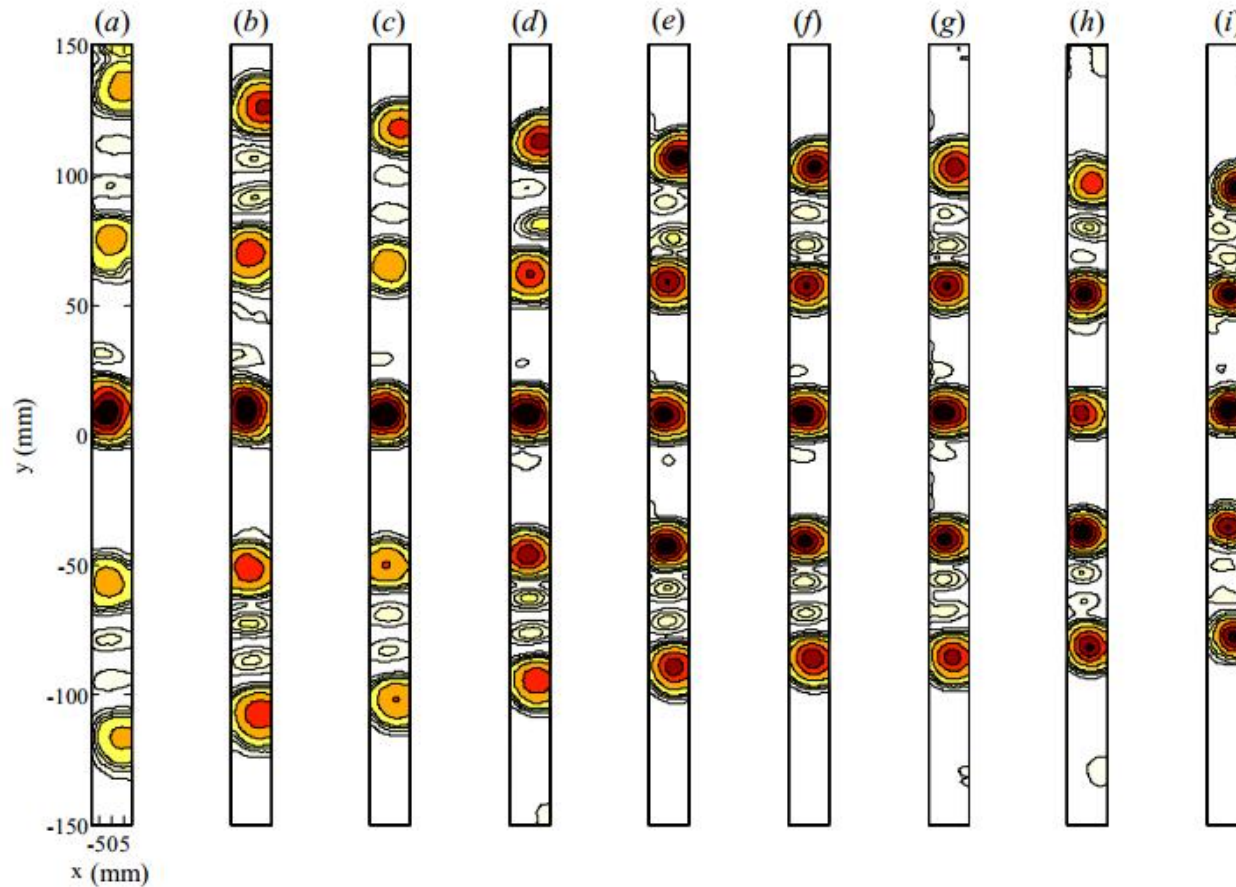


• *Simulated Pattern*



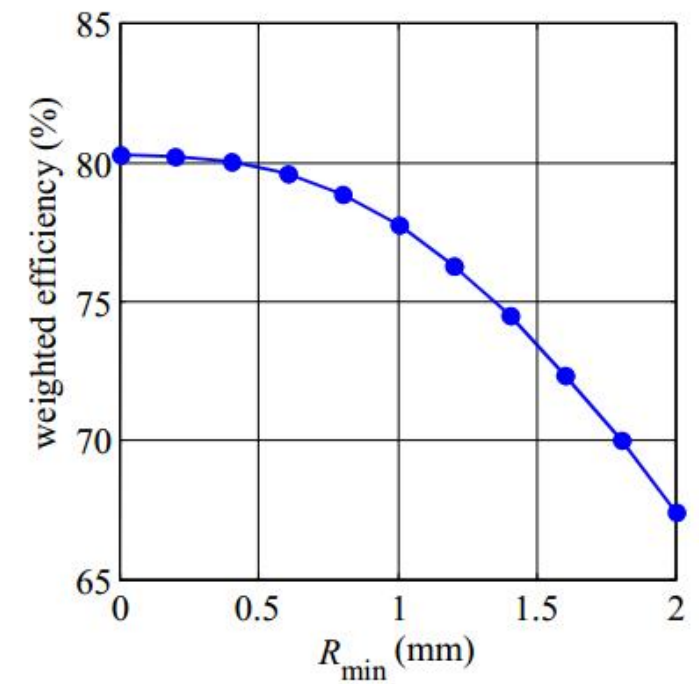
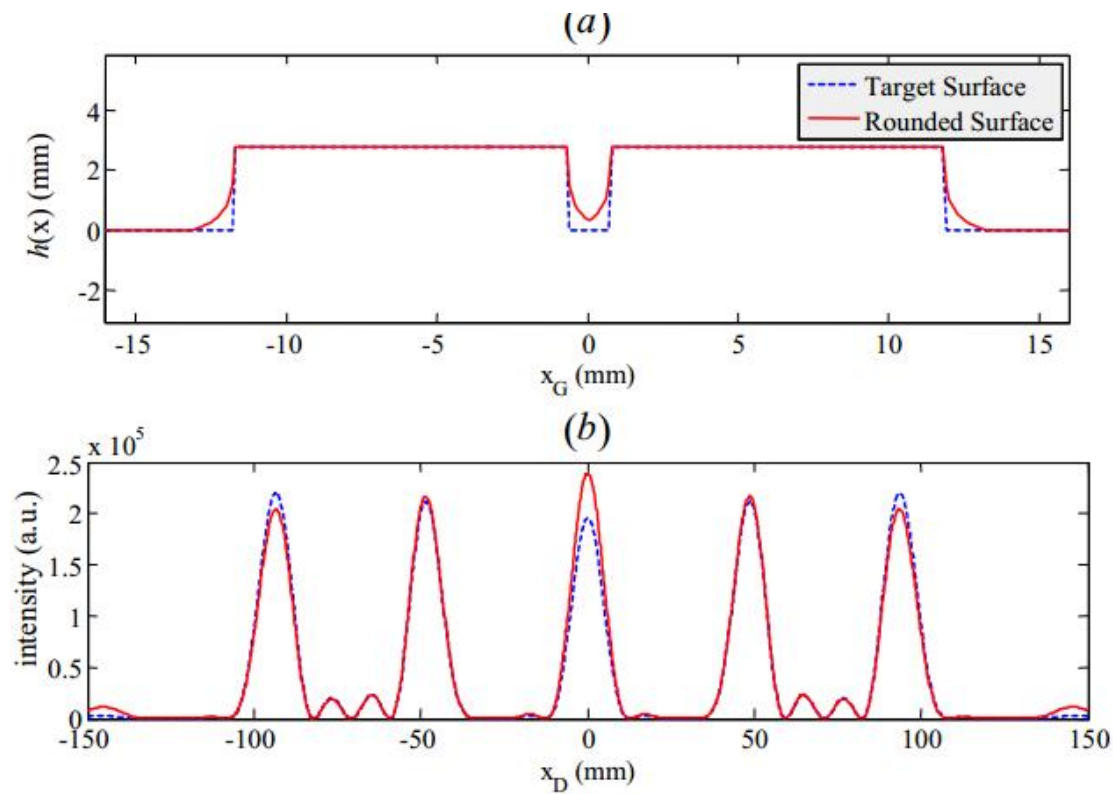
## Dammann Phase Grating Bandwidth effects

- *Dammann Grating (Binary) -*
- *Analyse off-axis configurations using MODAL.*



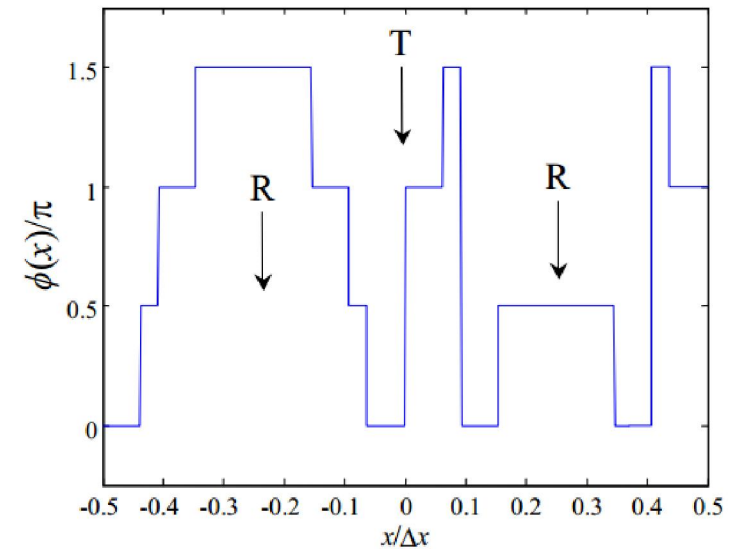
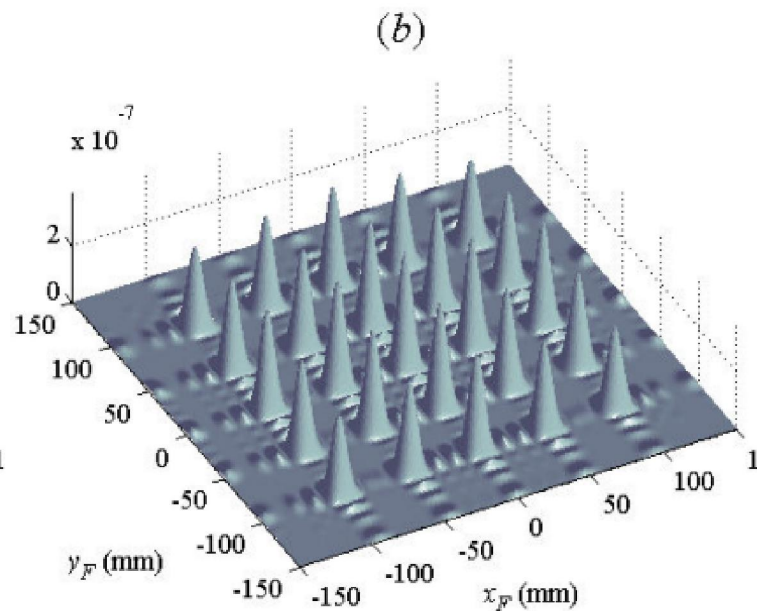
# Quasioptical Dammann Grating

- Tolerances



# Quasioptical Fourier Grating

- Higher efficiencies achievable

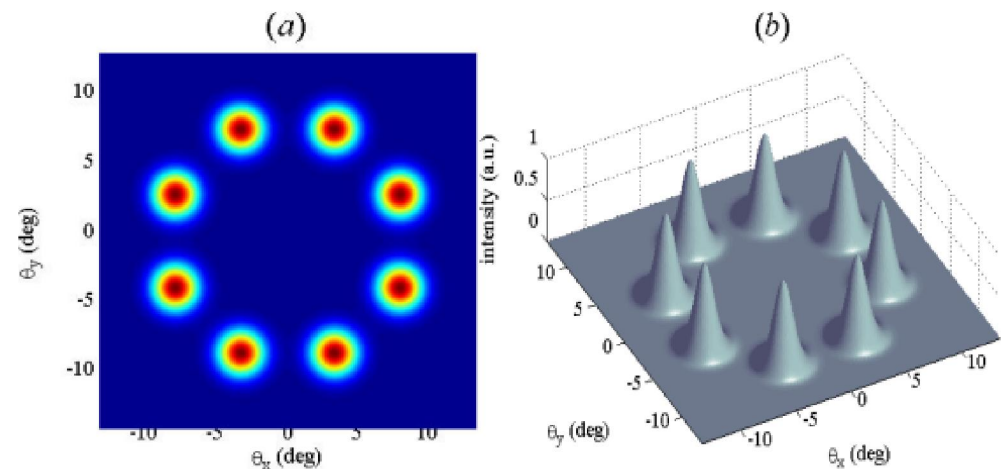
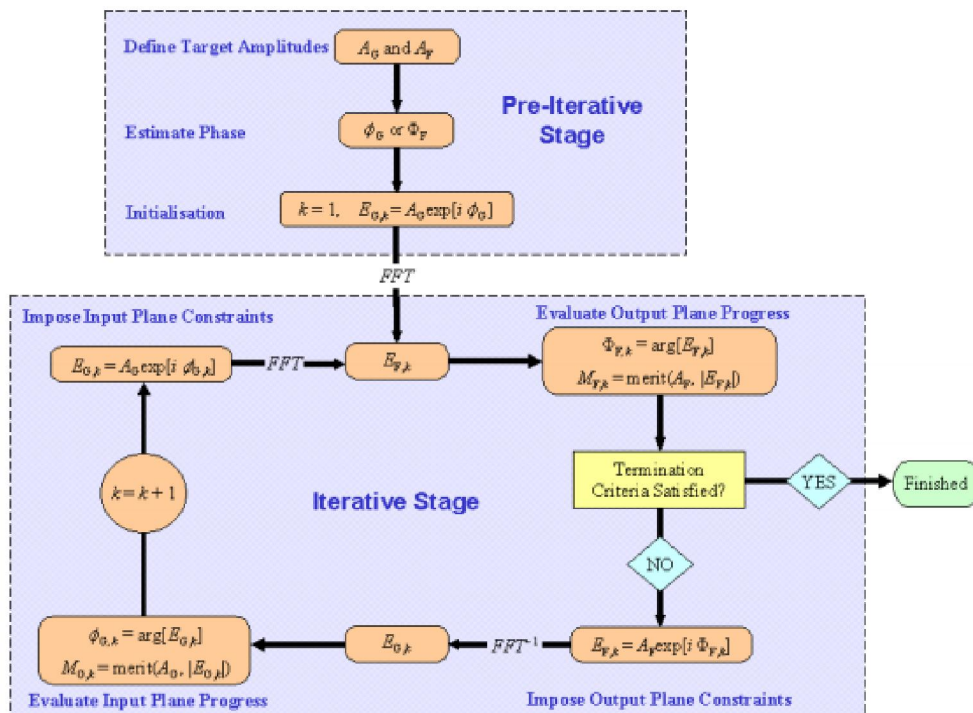


# fabrication steps	# phase levels ( $= 2^m$ )	step size, $\Delta\phi$ ( $= 2\pi/2^m$ )	Phase Levels $\{\phi_m\} = \{0, 1, 2, \dots, m\} \Delta\phi$
1	2	$\pi$	$\{0, 1\} \pi$
2	4	$\pi/2$	$\{0, 1, 2, 3\} \pi/2$
3	8	$\pi/4$	$\{0, 1, 2, 3, 4, 5, 6, 7\} \pi/4$



# Quasioptical Fourier Grating

- Bidirectional algorithms (Iterative Phase Retrieval) - the incident beam at the grating plane and the far-field diffraction intensity pattern.

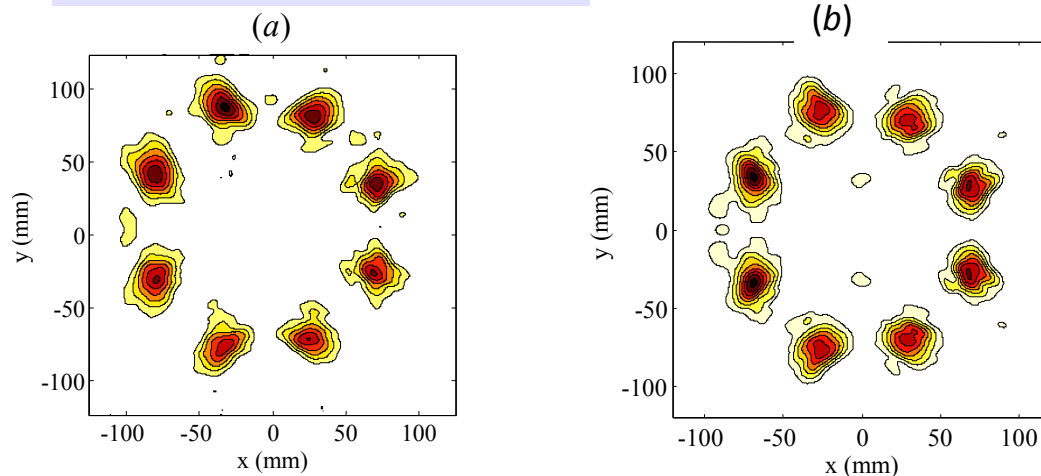
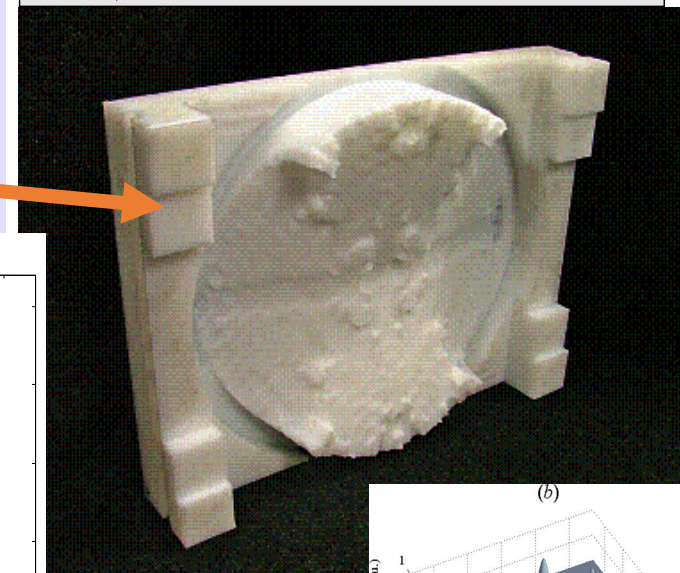
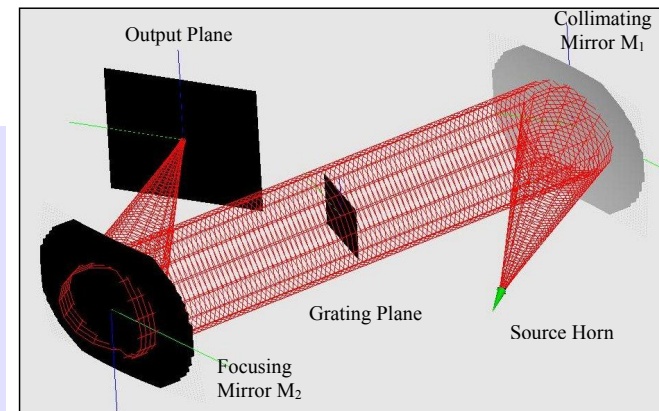


## Fourier Phase Gratings

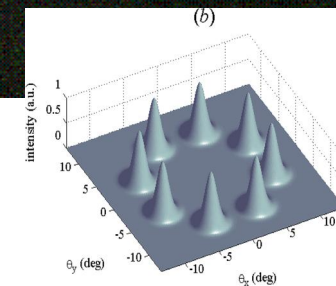
*For arrays without Cartesian symmetry design Fourier grating with continuously varying profile - (R. May at NUIM).*

*Fourier Phase Grating designed using an Iterative Phase Retrieval Algorithm - GBMs to propagate between planes*

*HDPE grating for circular array of 8 beams was manufactured and tested.*

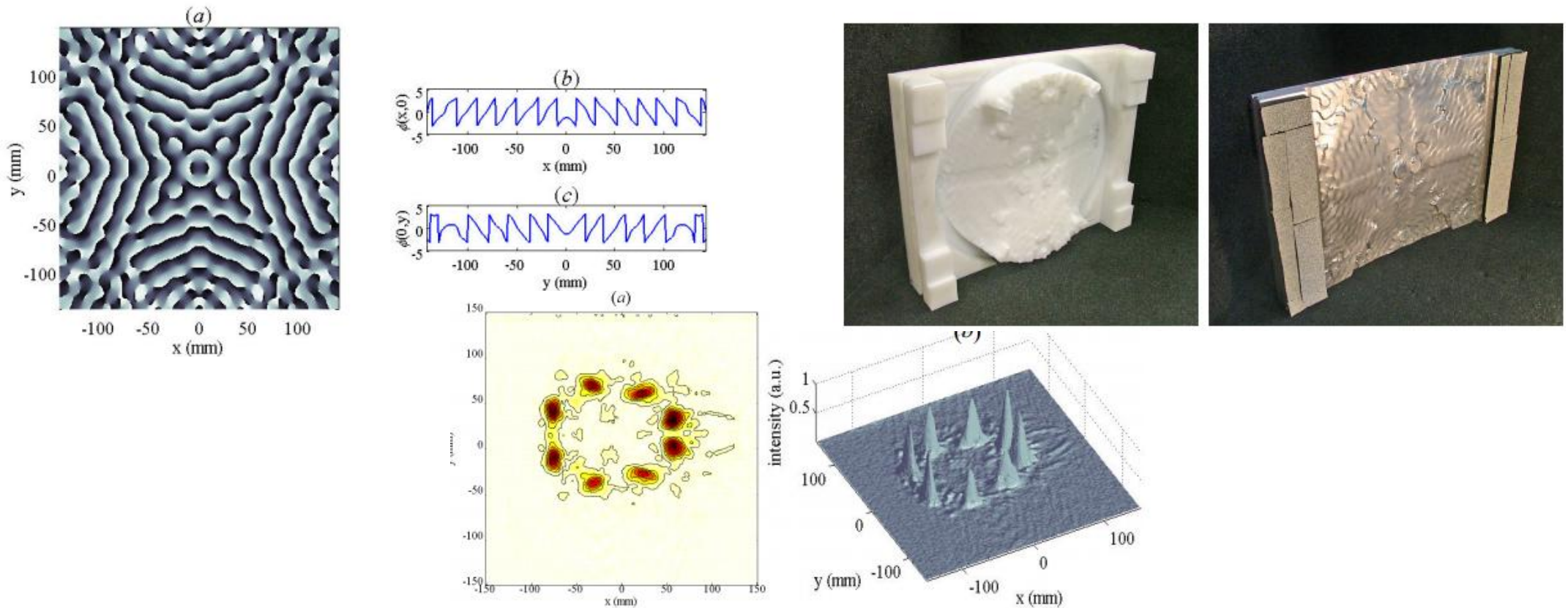


*Experimentally measured (a) compared with MODAL simulations (b) - (including aberrations & truncation effects of off-axis mirrors).*



# Quasioptical Fourier Grating

- Bidirectional algorithms (Iterative Phase Retrieval) - the incident beam at the grating plane and the far-field diffraction intensity pattern.



# New Measurement Facilities at Maynooth

- W Band VNA test facility
- Recently funded for multiplier heads from 500 GHz to 1100 GHz
- We welcome anyone interested in testing or having components or systems tested

